**Evolution of Cooperation: The Analysis of the Case Wherein a Different Player Has a Different Benefit and a Different Cost**

Letters on Evolutionary Behavioral Science

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**Supplementary file**

**Appendix A**

*Proof for (1)*

We consider the game between a (*c*1, *b*1, *f* (*l*), 0) and (*c*2, *b*2, *f* (*l*), 0). We define *x* as the expected total payoff by an individual playing a (*c*1, *b*1, *f* (*l*), 0) in a game with two (*c*1, *b*1, *f* (*l*), 0)s and *y* as the expected total payoff by an individual playing a (*c*2, *b*2, *f* (*l*), 0) in a group consisting of one (*c*1, *b*1, *f* (*l*), 0) and one (*c*2, *b*2, *f* (*l*), 0).

We can then determine the condition under which a (*c*1, *b*1, *f* (*l*), 0) is an ESS against an invasion of (*c*2, *b*2, *f* (*l*), 0); we can show that the ESS condition will hold only if

$x>y$. (A. 1)

Now after algebraic calculations (see Sigmund, 2010), we have

$x=-\frac{\left(1-w\right)+wf\left(0\right)}{1-w}\frac{1+w\left[f\left(b\_{1}\right)-f\left(0\right)\right]}{1-\left[f\left(b\_{1}\right)-f\left(0\right)\right]^{2}w^{2}}c\_{1}+\frac{\left(1-w\right)+wf\left(0\right)}{1-w}\frac{1+w\left[f\left(b\_{1}\right)-f\left(0\right)\right]}{1-\left[f\left(b\_{1}\right)-f\left(0\right)\right]^{2}w^{2}}b\_{1}$ (A. 2)

$y=-\frac{\left(1-w\right)+wf\left(0\right)}{1-w}\frac{1+w\left[f\left(b\_{1}\right)-f\left(0\right)\right]}{1-\left[f\left(b\_{1}\right)-f\left(0\right)\right]\left[f\left(b\_{2}\right)-f\left(0\right)\right]w^{2}}c\_{2}+\frac{\left(1-w\right)+wf\left(0\right)}{1-w}\frac{1+w\left[f\left(b\_{2}\right)-f\left(0\right)\right]}{1-\left[f\left(b\_{1}\right)-f\left(0\right)\right]\left[f\left(b\_{2}\right)-f\left(0\right)\right]w^{2}}b\_{1}$ (A. 3)

Using (A. 1)–(A. 3), the ESS condition will hold only if (1). This is the end of the proof.

**Appendix B**

We consider the case wherein the game is played by individuals whose error rates are different in what follows.

Here, we consider the game between (*c*, *b*, *f* (*l*), $μ\_{1}$) and (*c*, *b*, *f* (*l*), $μ\_{2}$). In this case, after algebraic calculation (see Sigmund, 2010), we can know that when an (*c*, *b*, *f* (*l*), $μ\_{1}$) player encounters (*c*, *b*, *f* (*l*), $μ\_{1}$), (*c*, *b*, *f* (*l*), $μ\_{1}$) gets

$(b-c)\frac{\left(1-w\right)+wf\left(0\right)}{1-w}\frac{\left(1-μ\_{1}\right)}{1-\left[f\left(b\right)-f\left(0\right)\right]w\left(1-μ\_{1}\right)}$ (B. 1)

Similarly, when an (*c*, *b*, *f* (*l*), $μ\_{2}$) player encounters (*c*, *b*, *f* (*l*), $μ\_{1}$), (*c*, *b*, *f* (*l*), $μ\_{2}$) gets

$\frac{\left(1-w\right)+wf\left(0\right)}{1-w}\frac{1}{1-\left[f\left(b\right)-f\left(0\right)\right]^{2}w^{2}\left(1-μ\_{1}\right)\left(1-μ\_{2}\right)}(b\left(1-μ\_{1}\right)\left(1+w\left[f\left(b\right)-f\left(0\right)\right]\left(1-μ\_{2}\right)\right)-c\left(1-μ\_{2}\right)\left(1+w\left[f\left(b\right)-f\left(0\right)\right]\left(1-μ\_{1}\right)\right))$ (B. 2)

Using (B. 1) and (B. 2), we can know that the condition under which (*c*, *b*, *f* (*l*), $μ\_{1}$) is stable against an invasion of (*c*, *b*, *f* (*l*), $μ\_{2}$) is given as

$(wb\left(1-μ\_{1}\right)\left[f\left(b\right)-f\left(0\right)\right]-c)\left(μ\_{1}-μ\_{2}\right)<0.$ (B. 3)

Assuming that $f\left(b\right)>f\left(0\right)$,

$\left(μ\_{1}-μ\_{2}\right)\left(μ\_{1}-μ^{\*}\right)>0.$ (B. 4)

where $μ^{\*}=1-\frac{c}{wb\left[f\left(b\right)-f\left(0\right)\right]}$

This means that when the error rate is smaller than the critical value ($μ^{\*}$), a strategy which has a smaller error rate than the resident strategy can invade, while when the error rate is larger than the critical value ($μ^{\*}$), a strategy which has a larger error rate than the resident strategy can invade. Thus, the error rate goes to 0 or 1, which depends on the initial state. This result can be regarded similar with special case 3. Note that when $c>wb\left[f\left(b\right)-f\left(0\right)\right]$ is met, $μ^{\*}<0$ holds true; hence, the error rate goes to 1, irrespective of the initial state.

**References**

Sigmund, K. (2010). The calculus of selfishness. Princeton: Princeton University Press. (doi: 10.1515/9781400832255)