Evolution of Cooperation: The Analysis of the Case Wherein a Different Player Has a Different Benefit and a Different Cost

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The existence of cooperation demands explanation in terms of natural selection. Prisoner's dilemma is a framework often used when studying the evolution of cooperation. In prisoner's dilemma, most previous studies consider the situation wherein an individual who cooperates will give an opponent an amount b at a personal cost of c, where $b > c > 0$ while an individual who defects will give nothing. This model setting is convenient; however, previous studies have not considered the case wherein a different player has a different benefit and different cost while in reality, it is natural to consider that a different player has a different benefit and different cost. Here, we raise the following question: Taking that a different individual has a different benefit and a different cost into consideration, what strategy is likely to evolve? In this paper, we focus on the direct reciprocity and analyze the case wherein each player follows the same probability (or reaction) function and but different players have a different benefit and cost. We obtain the condition for the evolution in the general case. And in addition, we reveal that under a specific condition as the interaction repeats longer and the benefit-to-cost ratio is larger and the cooperating probability is more sensitive to the benefit the opponent provides, the establishment of cooperation is more likely.

Keywords

Introduction

The existence of cooperation demands explanation in terms of natural selection (Hamilton, 1964; Trivers, 1971). Prisoner's dilemma is a framework often used when studying the evolution of cooperation. In prisoner's dilemma, most previous studies consider the situation wherein an individual who cooperates will give an amount b to an opponent while paying a personal cost of c where $b > c > 0$ while an individual who defects will not give anything.

This model setting is convenient; however, in reality, the cooperation level is continuous (Doebeli, Hauert, & Killingback, 2004; Killingback, Doebeli, & Knowlton, 1999; Nakamaru & Dieckmann, 2009; Shimao & Nakamaru, 2013; Wahl & Nowak, 1999a,b). Some previous studies have made a research on the evolution of cooperation in the case wherein individuals can cooperate continuously.

However, these previous studies considered the situation wherein a different individual has the same benefit cost function, while considering the situation wherein a different individual has a different benefit and a different cost is reasonable.

This paper considers the case wherein each player follows the same probability (or reaction) function and but different players have a different benefit and cost. Here, we raise the following question: Taking that a different individual has a different benefit and a different cost into consideration, what strategy is likely to evolve? We focus on the direct reciprocity (see Kurokawa, 2016a,b,c; Kurokawa, in press; Kurokawa & Ihara, 2009; Kurokawa, Wakano, & Ihara, 2010; Trivers, 1971 for papers on direct reciprocity) in this paper.

Model

Assume that individuals are paired randomly and interact, and that age structure is absent (see Li, Kurokawa, Giaimo, & Traulsen, 2016 for a study dealing with age structured population). We consider the repeated prisoner's dilemma game in which two individuals have to either cooperate or defect in each round. The probability of the individuals' interacting over t times in a given pair is w^t, where $0 < w < 1$ holds true. This assumption leads to that the expected number of interactions is given as $1/(1 - w)$.

We consider strategies given by $(c, b, f(l), \mu)$. If a receiver gets an amount b, where $b > 0$, while paying a cost c, where $c > 0$. And let $f(l)$ be the probability to cooperate in the following round when the opponent gave benefit l to the focal player in the previous round, where $0 \leq f(l) \leq 1$ (see e.g., Roberts & Sherratt, 1998; Wahl & Nowak, 1999a,b for papers studying the strategy which decided what to do next based on the benefit the opponent provided in the previous round). Note that $(c, b, f(l), \mu)$ cooperates with probability $f(0)$ in the following round when the opponent defected in the previous round. Even when $(c, b, f(l), \mu)$ attempts to cooperate, $(c, b, f(l), \mu)$ fails to do so with probability μ, where $0 \leq \mu < 1$ (Kurokawa, 2016c; May, 1987). $(c, b, f(l), \mu)$ player always tries to cooperate in the first round. This can be regarded as an extension of earlier models (e.g., Axelrod & Hamilton, 1981).

Result

Here, we consider the game between $(c_1, b_1, f(l), 0)$ and $(c_2, b_2, f(l), 0)$. After algebraic calculation (see Appendix A in supplementary file for the detailed calculation), it is shown that the condition for $(c_1, b_1, f(l), 0)$ to be stable against the invasion by $(c_2, b_2, f(l), 0)$ is given as

$$(1 - m_1 m_2 w^2)c_1 - (1 - m_1 m_1 w^2)c_2 < w(m_1 - m_2)b_1, \quad (1)$$

where $m_1 \equiv f(b_1) - f(0)$, $m_2 \equiv f(b_2) - f(0)$.

It can be shown that it is likely that (1) is violated when c_2 is small, which indicates that the strategy with a small cost is likely to invade.

This is the condition in the general case. In the following three sections, we consider special cases and we investigate what strategy is successful.

Special case 1 (the case wherein c is independent of strategies)

In this section, we consider the case wherein cost is independent of strategies. Substituting $c_1 = c_2$ into (1), (1) becomes

$$(b_1 - m_1 w c_1)(f(b_1) - f(b_2)) > 0. \quad (2)$$

Here, we assume that $b_1 > c_1$ is met. Under this assumption, (2) becomes

$$f(b_1) > f(b_2). \quad (3)$$

If a strategy has the benefit (b) whose $f(b)$ is larger than $f(b)$ of the invasion strategy, the resident strategy ($c_1, b_1, f(l), 0$) is evolutionarily stable against the invasion by ($c_2, b_2, f(l), 0$). Since inequality (3) does not contain $f(0)$ this result is not affected by the value of $f(0)$, which indicates that the response to those who defected in the previous round does not affect the outcome of the evolution.

Special case 2 (the case wherein $f(b_1) = f(b_2)$)

In this section, we consider the case wherein $f(b_1) = f(b_2)$. In this situation, a player's probability of cooperating in the next round depends only qualitatively on whether an opponent player cooperates at all, rather than quantitatively on how much the player provides. Substituting $f(b_1) = f(b_2)$ into (1), (1) becomes

$$c_1 < c_2. \quad (4)$$

This means that if the invasion strategy has a larger cost than the resident strategy, then the invasion strategy has a lower payoff than the resident strategy and fails to invade. On the other hand, if the invasion strategy has a smaller cost than the resident strategy, then the invasion strategy has a higher payoff than the resident strategy and succeeds in invading. The ultimate outcome of the evolution is that a strategy which pays little cost to the opponent spreads over the population (see also Doebeli & Knowlton, 1998). We also note that $b_1 = b_2$ is a sufficient condition for $f(b_1) = f(b_2)$; hence, the condition for $(c_1, b_1, f(l), 0)$ to be stable against the invasion by $(c_2, b_2, f(l), 0)$ in the case wherein $b_1 = b_2$ is met is given as (4).

Special case 3 (the case wherein $m_1/b_1 = m_2/b_2 = t$ is satisfied, and there exist only strategies which satisfy $b_1/c_1 = b_2/c_2 = k$)

In this section, we consider the case wherein $m_1/b_1 = m_2/b_2 = t$ (where $t \geq 0$) is satisfied, and there exist only strategies which satisfy $b_1/c_1 = b_2/c_2 = k$ (where $k > 1$).

If $t > 0$ is met, using these equations, (1) becomes

$$(c_1 - c_2)(c_1 - c^*) > 0, \quad (5)$$

where $c^* = 1/(wk^2 t)$.

This means that if the cost the resident strategy has is smaller than the critical value (c^*), the strategy with a larger cost cannot invade the resident strategy, while the strategy with a smaller cost can invade the resident strategy. On the other hand, if the cost the resident strategy has is larger than the critical value (c^*), the strategy with a smaller cost cannot invade the resident strategy, while the strategy with a larger cost can invade the resident strategy. The critical value is small when the parameters (w, k, and t) are large, which indicates that as the interaction repeats longer and the benefit-to-cost ratio is larger and the cooperating probability is more sensitive to the benefit the opponent provides, the basin of attraction for cooperation is larger and the establishment of cooperation is more likely. This result is not affected by the value of $f(0)$, which indicates that the response to those who defected in the previous round does not affect the outcome of the evolution. However, in either way, in order to make the initial evolution of cooperation possible, some mechanism must be required.

Figure 1 illustrates the evolution of strategies. The parameters (c and b) are indexed, respectively, on the horizontal and vertical axes. The circle indicates unstable internal equilibrium point. If the state is at the lower left than the unstable internal equilibrium point, the state goes to lower left. On the contrary, if the state is at the upper right than the unstable internal equilibrium point, the state goes to upper right.
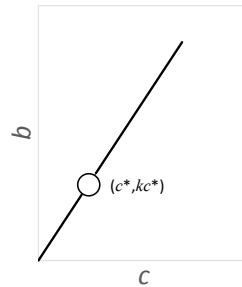

Figure 1. Strategy evolution.

In the case wherein $t = 0$ is met, $f(b)$ is independent of strategies and we dealt this case in special case 2. Hence, please see special case 2.

Discussion

This paper analyzes the case wherein each player follows the same probability (or reaction) function and but different players have a different benefit and cost (i.e., cost and benefit are not exogenous). And we obtained the condition for the evolution of strategies in the general model. In addition, we consider three specific cases.

Special case 3 considers the case wherein the cost (the benefit) is various while the benefit-to-cost ratio is constant, irrespective of cost. On the other hand, there are previous studies which consider the case wherein players are error-prone (e.g., Kurokawa, 2016c; May, 1987). When taking an average over long time, the effect of strategies whose benefit is smaller on their payoff (fitness) may be similar with the effect of strategies who fails to cooperate

with more probability on their payoff (fitness). Are these two topics qualitatively the same? It can be shown that these two topics are qualitatively the same. Please see Appendix B for the detailed discussion.

Special case 3 examines the case wherein the benefit-to-cost ratio is constant, irrespective of cost. On the contrary, the benefit-to-cost ratio decreases as the cost increases in some animals (e.g., *Corcorax melanorhamphos*) (Le Galliard, Ferrière, & Dieckmann, 2003). In such cases, what strategy is likely to evolve? Further study on this issue is needed.

S. Kurokawa (unpublished data) considered the strategy which has revealed that strategies which cooperate more when the actor cooperated in the previous round than when the actor defected in the previous round (strategies with stubbornness) are more successful. On the contrary, Killingback & Doebeli (2002) considered the payoff-based strategies (strategies with contra-stubbornness), which cooperate more when the actor defected in the previous round than when the actor cooperated in the previous round and stated that such a payoff-based strategy is beneficial for its evolution. Thus, some previous studies use the information about what the actor did in the previous round. On the other hand, the strategy in this paper does not use the information about what the actor did in the previous round. It might be interesting to examine the strategy which refers to the actor's past move. Further study on this issue is required.

This paper only considers the interaction by two individuals. However, some animals including humans interact within a group consisting of more than two individuals. In order to analyze such a group-wise interaction, we have to extend this two player games to *n*-player games (Deng, Li, Kurokawa, & Chu, 2012; Kurokawa & Ihara, 2009, 2013; Kurokawa et al., 2010). Further study on this issue is needed.

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